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# Application of Feedback Linearization Method in a Digital Restructurable Flight Control System

Yoshimasa Ochi*
*National Defense Academy, Yokosuka, Kanagawa, 239 Japan*

**The proposed control system is applied to aircraft failures occurring with the control effectors or airframe. The failures are identified as parameter changes in the six-degree-of-freedom nonlinear equations of motion by the recursive least-squares algorithm. The control parameters are updated using the latest estimated system parameters. In order to allow one to use a digital computer in implementations, a discrete-time servo controller is designed for the actuators of the control surfaces and engine. The reference inputs are given by the continuous-time control law that is derived without considering actuator or engine dynamics. The frequency-dependent optimal regulator is employed in the design of the servo controller to eliminate input fluctuation. Performance of the control system is demonstrated through computer simulation using the nonlinear model of an aircraft that has half its right wing broken off.**

## Introduction

WHILE today's advanced technologies are producing high-performance aircraft both for military and commercial use, the aircraft involved have larger and more complicated systems. Automatic in-flight control systems have been increasingly introduced. As a result, supervising or monitoring the systems is taking the place of handling the aircraft as the pilot's primary task. This often makes it difficult for pilots to cope with failures or anomalous situations adequately. Such aircraft, more and more expensive, are required to survive or remain available even if they suffer failures or damage; this is especially true for military planes.

Thus, the importance of failure accommodation has come to be realized. Studies on the problem started in large with the concept of a restructurable or reconfigurable flight control system (RFCS), which was proposed at a NASA workshop in 1982. Many papers have been reported since then.[1-6] Among them is the study of self-repairing FCS,[3] which represents a simple and practicable approach. It was evaluated by not only numerical simulation, but also motion-based simulation.[4] Furthermore, flight tests on the F-15[5] were carried out. However, from the point of view of controller design, the studies are based on linearized mathematical models and linear control theories. Although linear controllers are known to be useful in practice even in some nonlinear environments, failures often cause severe nonlinearity, such as highly coupled motions or large motions apart from the nominal trim point. In those situations, the linearized models can no longer describe the impaired aircraft dynamics well so that the RFCS may fail to accommodate failures.

The author previously proposed a design of continuous-time (CT) RFCS using the feedback linearization method (FLM).[7] The first feature of RFCS is that failures are identified as parameter changes in the six-degree-of-freedom (6-DOF) nonlinear equations of motion by the recursive least-squares algorithm, and the control parameters are updated using the latest estimated parameters. Therefore, RFCS has the potential to accommodate failures that considerably change the characteristics of the aircraft dynamics, including stability, as well as those of the control effectors. The second feature of RFCS is that the parameters are identified using generic inputs, the number of which is smaller than that of actual inputs, i.e., control surfaces. The control law is also determined for generic inputs. It is desirable to have many inputs take advantage of functional redundancy.[6] However, the more inputs there are, the more parameters have to be identified, which means that it requires more time and computation efforts to complete the identification. By using generic inputs, the number of system parameters reduces, so identification time does also.

There are three new points in this paper. The first one is that the RFCS is a discrete-time (DT) system. Although feedback linearization[8,9] is a CT design method, using a digital computer is inevitable in implementation. Therefore, it is necessary to redesign the RFCS as a DT controller. The approach employed is to design a digital servo controller for the actuators and engine. Their dynamics are assumed to be modeled as a first-order system. The reference inputs to it are the CT control laws that give the control surface deflections and engine output. Note that the CT control laws are obtained without considering the actuator and engine dynamics. The frequency-dependent optimal regulator (FDOR)[10] is incorporated into the servo controller. The FDOR prevents the RFCS from generating high-frequency inputs to the actuators or engine. Such inputs can excite the high-frequency modes and lead to instability.

The second point is that the RFCS is more robust in nonlinear situations than the system of Ref. 7. In general, in order to control the pitch and roll angles by the FLM, their third derivatives must be computed for the following reasons. The first is that the state equations for the pitch and roll angles include no control effectors such as control surfaces. The other is that the actuator dynamics are considered. Since computing the third derivatives makes the design complicated, an approximate method was employed to find the CT control law for those angles in Ref. 7. However, the approximation becomes incorrect as the angles grow large. Sometimes, the RFCS results in poor control performance. In contrast, the proposed RFCS is free from such a problem. In the design, first the CT control law is obtained by applying the FLM in a straightforward manner without considering the actuator dynamics, where at most the second derivatives are required. And then, the DT servo controller is designed for the actuator. The reference input is supplied by the CT control law. Thus, the combination of the CT control law and DT servo controller makes the RFCS simpler, more robust, and digital.

The third point is that the number of outputs is less than that of the inputs in FLM design, where a pseudoinverse is used to obtain the control law.[3] Whereas the four outputs, angle of attack, pitch angle, sideslip angle, and roll angle, are controlled in Ref. 7, the sideslip angle is not covered here. It is true that in such situations as minor failures or smaller maneuvers, controlling the sideslip angle along with other angles results in

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*Research Associate, Department of Aerospace Engineering, 1-10-20 Hashirimizu. Member AIAA.

better output responses. Otherwise, it may sometimes result in catastrophic circumstances, probably because controlling the sideslip angle as well is too demanding to an aircraft whose control power is limited.

In this paper, sensors and computers on the aircraft are assumed to be normal, and sensor noises are not considered.

## Description of Aircraft Model

It is assumed that a mathematical model of an aircraft including the actuator and engine dynamics that are approximated by the first-order system is given by

$$\text{S1:}\quad \dot{u} = A_1(X) + \boldsymbol{B}_{11}(X)\boldsymbol{\delta} + B_{12}T_h \tag{1}$$

$$\dot{T}_h = (-T_h + T_{hc})/T_e \tag{2}$$

$$\text{S2:}\quad \dot{X}_2 = \boldsymbol{A}_2(X) + B_2(X)\boldsymbol{\delta} \tag{3}$$

$$\dot{\boldsymbol{\delta}} = \Lambda(-\boldsymbol{\delta} + \boldsymbol{\delta}_c) \tag{4}$$

In the equations, the state vector equals $X_a = [X^T, U^T]^T$, where $X = [u, X_2^T]^T$ and $U = [\delta^T, T_h]^T$, and the control vector is $U_c = [\delta_c^T, T_{hc}]^T$. Definitions of the symbols are as follows. $X_2 = [w, q, \theta, v, r, p, \phi]^T$, $\delta = [\delta_{hL}, \delta_{hR}, \delta_{aL}, \delta_{aR}, \delta_{cL}, \delta_{cR}, \delta_r]^T$, where $u$ = speed along the $X$ axis (m/s), $w$ = speed along the $Z$ axis (m/s), $q$ = pitch rate (rad/s), $\theta$ = pitch angle (rad), $v$ = speed along the $Y$ axis (m/s), $r$ = yaw rate (rad/s), $p$ = roll rate (rad/s), and $\phi$ = roll angle (rad). The $\delta_{hL}$ and so on are control surface deflection angles, where $h$ = horizontal tail, $a$ = aileron, $c$ = canard, $r$ = rudder, $L$ = left surface, and $R$ = right surface. The $\delta_c = [\delta_{hLc}, \delta_{hRc}, \delta_{aLc}, \delta_{aRc}, \delta_{cLc}, \delta_{cRc}, \delta_{rc}]^T$ represents command inputs to the surface actuators. $T_h$ is thrust and $T_{hc}$ the throttle input. $A_1(X) \in R^1$, $\boldsymbol{B}_{11}(X) \in R^{1\times 7}$, and each element of $\boldsymbol{A}_2(X) \in R^7$ and $B_2(X) \in R^{7\times 7}$ has a linear combination of constant parameters and known functions of $X$ (see the Appendix). Finally, $\Lambda = \text{diag}\{1/T_i\}$, $T_i$ being a time constant of the $i$th actuator ($i = 1, \ldots, 7$) and $T_e$ a time constant of the engine.

As seen from Eqs. (1) and (3), it is assumed that the forces and moments produced by the thrust are negligible, except for the force along the $X$ axis. Since $u$ changes slowly compared with other states of $X_2$, it is expected that the change of $u$ has an insignificant effect on Eq. (3). From the foregoing observation, the system S2 represented by Eqs. (3) and (4) can be approximately regarded as being decoupled from the system S1 by Eqs. (1) and (2). Then, since $\delta$ can be determined independently of Eq. (1), S1 could be defined as a single-input single-output system. In S1 the input is the thrust and the output the speed.

In the FLM, at least the same number of inputs as that of outputs are requisite to control the outputs. Let us choose three outputs (angle of attack, pitch angle, and roll angle). Then let us introduce the control distributor (CD) and generic inputs, $\boldsymbol{\delta}_G = [\delta_{\text{lng}1}, \delta_{\text{lng}2}, \delta_{\text{lat}}, \delta_{\text{dir}}]^T \in R^4$. The $\boldsymbol{\delta}_G$ is defined by

$$\boldsymbol{\delta} = P\boldsymbol{\delta}_G \tag{5}$$

where $P \in R^{7\times 4}$ is a constant CD matrix.

Mathematically, $P$ can be chosen arbitrarily as long as it has full rank. In this paper, however, $P$ was selected as Eq. (32) by the following rules:

1) The generic inputs should correspond to the actual control surfaces. That is, $\delta_{\text{lng}1}$ and $\delta_{\text{lng}2}$ correspond to the longitudinal control surfaces such as elevators, $\delta_{\text{lat}}$ to the lateral ones such as ailerons, and $\delta_{\text{dir}}$ to a directional one such as a rudder.

2) The magnitudes of the elements should be in proportion to the deflection limits of the related surfaces in principle.

The reason why the two longitudinal inputs $\delta_{\text{lng}1}$ and $\delta_{\text{lng}2}$ are chosen is that two longitudinal outputs, $\alpha$ and $\theta$, are selected. In this way, deflection angles of the actual surfaces are expected to be reasonable and kept from saturation unless the control demands are extremely severe.

Substituting Eq. (5) into Eqs. (1) and (3) yields, respectively,

$$\dot{u} = A_1(X) + \boldsymbol{B}_{11G}(X)\boldsymbol{\delta}_G + B_{12}T_h \tag{6}$$

$$\dot{X}_2 = \boldsymbol{A}_2(X) + B_{2G}(X)\boldsymbol{\delta}_G \tag{7}$$

where

$$\boldsymbol{B}_{11G}(X) = \boldsymbol{B}_{11}(X)P \in R^{1\times 4}$$

and

$$B_{2G}(X) = B_2(X)P \in R^{7\times 4}$$

Equations (6) and (7) can also be written as

$$\dot{X} = \boldsymbol{A}(X) + B_G(X)U_G \tag{8}$$

where $\boldsymbol{A}(X) = [A_1(X), \boldsymbol{A}_2(X)^T]^T$,

$$B_G(X) = \begin{bmatrix} \boldsymbol{B}_{11G}(X) & B_{12} \\ B_{2G}(X) & 0 \end{bmatrix}$$

and $U_G = [\delta_G^T, T_h]^T$.

The actuator dynamics for generic inputs are defined by

$$\dot{\boldsymbol{\delta}}_G = \Lambda_G(-\boldsymbol{\delta}_G + \boldsymbol{\delta}_{Gc}) \tag{9}$$

where $\boldsymbol{\delta}_{Gc} \in R^4$ is a command generic input vector defined by

$$\boldsymbol{\delta}_c = P\boldsymbol{\delta}_{Gc} \tag{10}$$

$\Lambda_G = I_4/T_{\max}$ ($I_i$ being an $i\times i$ identity matrix), and $T_{\max}$ is the largest time constant of the actuators. We refer to the actuator described by Eq. (9) as the imaginary actuator.[7]

## Continuous-Time RFCS

The CT control system described in this section is a part of the digital RFCS, i.e., a generator of reference inputs to the digital servo controller designed for the actuators and engine.

Since the control objective of the RFCS is to settle the aircraft motion at a trim point, the outputs to be controlled are chosen as $Y = [u, Y_1, Y_2^T]^T$, where $Y_1 = \alpha$ $[= \tan^{-1}(w/u)]$ and $Y_2 = [\theta, \phi]^T$. By differentiating $Y$, the following equations are obtained:

$$\dot{u} = A_1(X) + \boldsymbol{B}_{11G}(X)\boldsymbol{\delta}_G + B_{12}T_h \tag{11}$$

$$\dot{Y}_1 = A'_{21}(X) + \boldsymbol{B}'_{2G1}(X)\boldsymbol{\delta}_G \tag{12}$$

$$\dot{Y}_2 = \boldsymbol{A}'_2(X_2) \tag{13}$$

where

$$\boldsymbol{A}'_2(X_2) = [A_{23}(X_2), A_{27}(X_2)]^T$$

$$A'_{21}(X) = [A_{21}(X) - A_1(X)\tan\alpha]\cos^2\alpha/u$$

and

$$\boldsymbol{B}'_{2G1}(X) = [\boldsymbol{B}_{2G1}(X) - \boldsymbol{B}_{11G}(X)\tan\alpha]\cos^2\alpha/u$$

$A_{2i}(X_2)$ and $\boldsymbol{B}_{2Gi}(X)$ are the $i$th element of $\boldsymbol{A}_2(X)$ and the $i$th row of $B_{2G}(X)$, respectively. Since S2 can be regarded as being decoupled from S1, first $\boldsymbol{\delta}_G$ is determined for S2 [Eqs. (12) and (13)] and then $T_h$ obtained from Eq. (11).

Equation (13) includes no control input, but by differentiating it yields

$$\ddot{Y}_2 = \boldsymbol{A}''_2(X) + B'_{2G}(X)\boldsymbol{\delta}_G \tag{14}$$

where $\boldsymbol{A}''_2(X) = \{\partial\boldsymbol{A}'_2(X_2)/\partial X_2^T\}\boldsymbol{A}_2(X)$, $B'_{2G}(X) = \{\partial\boldsymbol{A}'_2(X_2)/\partial X_2^T\}B_{2G}(X)$. By applying the FLM to Eqs. (12) and (14), the control law that makes $[Y_1, Y_2^T]^T$ follow the reference outputs $[Y_1^*, Y_2^{*T}]^T$ is given by

$$\delta_G = \begin{bmatrix} B'_{2G1}(X) \\ B'_{2G}(X) \end{bmatrix}^+ \left\{ -\begin{bmatrix} A'_{21}(X) \\ A''_2(X) \end{bmatrix} + \begin{bmatrix} G_1 y_1 \\ G_2 \eta \end{bmatrix} + \begin{bmatrix} \dot{Y}_1^* \\ \ddot{Y}_2^* \end{bmatrix} \right\} \tag{15}$$

where $y_1 = Y_1 - Y_1^*$, $\eta = [y_2^T, y_2'^T]^T$, $y_2 = Y_2 - Y_2^*$, $y_2' = \dot{y}_2 = \dot{Y}_2 - \dot{Y}_2^*$, and $+$ indicates the pseudoinverse. The pseudoinverse was first used to derive a reconfigurable control law in Ref. 3. $[B'_{2G1}(X)^T, B'_{2G}(X)^T]^T$ is assumed to have full rank ($=3$). In fact, by substituting Eq. (15) into Eqs. (12) and (14), they become, respectively,

$$\dot{y}_1 = G_1 y_1 \tag{16}$$

$$\dot{y}_2' = G_2 \eta \tag{17}$$

Provided that $G_1$ is given a negative number, then $y_1 \to 0$, i.e., $Y_1 \to Y_1^*$ as $t \to \infty$. As for $y_2$, from Eq. (12) and the definition of $y_2'$, the following augmented system is obtained.

$$\dot{\eta} = E\eta \tag{18}$$

where

$$E = \begin{bmatrix} 0 & I_2 \\ & G_2 \end{bmatrix} \in R^{4\times 4}$$

If $G_2 \in R^{2\times 4}$ is given a matrix that makes $E$ a stable one, then $\eta \to \mathbf{0}$ or $y_2 \to \mathbf{0}$ as $t \to \infty$.

Now that $\delta_G$ is found by Eq. (15), the first and second terms of Eq. (11) become known. Then, the control law for $u$ is given by

$$T_h = B_{12}^{-1}\{-A_1(X) - B_{11G}(X)\delta_G + G_3(u - u^*) + \dot{u}^*\} \tag{19}$$

where $G_3$ is a constant negative number and $u^*$ a reference output of $u$. In the same way as in Eq. (16), it is shown that $u$ converges to $u^*$.

Thus, by the control laws, Eqs. (15) and (19), the control objective $Y \to Y^*$ is attained.

*Adaptive Controller:* Parameters of the equations of motion, Eqs. (1) and (3), can reflect the effects that failures have on the aircraft dynamics. They are identified by the recursive least-squares method. The parameters of the control laws are updated at regular intervals using the latest estimated parameters of $A_1(X)$, $B_{11G}(X)$, and so on. From Eqs. (15) and (19), the adaptive-type control laws are given by

$$\delta_G = \begin{bmatrix} \hat{B}'_{2G1}(X) \\ \hat{B}'_{2G}(X) \end{bmatrix}^+ \left\{ -\begin{bmatrix} \hat{A}'_{21}(X) \\ \hat{A}''_2(X) \end{bmatrix} + \begin{bmatrix} G_1 y_1 \\ G_2 \eta \end{bmatrix} + \begin{bmatrix} \dot{Y}_1^* \\ \ddot{Y}_2^* \end{bmatrix} \right\} \tag{20}$$

$$T_h = \hat{B}_{12}^{-1}\{-\hat{A}_1(X) - \hat{B}_{11G}(X)\delta_G + G_3(u - u^*) + \dot{u}^*\} \tag{21}$$

where $\wedge$ means that the value is computed by replacing the true parameters with the estimated ones. The convergence properties of the output errors, $Y - Y^*$, are similar to those in Ref. 7.

## Discrete-Time Servo Controller

Since the state equations, Eqs. (1) and (3), are nonlinear, it is difficult to design the RFCS using the discrete-time mathematical model. Although the differential equations can be expressed approximately by the difference equations, the sampling rate must be sufficiently high. In this section, a description of the digital servo controller for the imaginary actuators and engine is given. By taking into account robust stability, the controller is compensated for by the frequency-dependent optimal regulator.[10] The FDOR can weight the frequency characteristics of inputs and outputs. If high-frequency inputs are weighted heavily, it is expected that the RFCS will be free from fluctuating or chattering inputs that have negative effects on hardware or may excite the unmodeled high-frequency modes such as flexible ones.

Let us consider the following frequency-dependent cost function for a linear system ($\dot{\bar{x}} = A\bar{x} + B\bar{u}$; $\bar{y} = C\bar{x}$; $A$, $B$, and $C$ are the constant matrices).

$$J = \left(\frac{1}{2\pi}\right) \int_{-\infty}^{+\infty} \{\bar{y}^* Q \bar{y} + \bar{u}^* R(j\omega)^* R(j\omega)\bar{u}\}\, d\omega \tag{22}$$

where $*$ denotes a complex conjugate transpose, $\omega$ is the angular frequency, and $j = (-1)^{1/2}$. $Q$ is a positive-definite symmetric matrix and $R(j\omega)^*R(j\omega)$ is a frequency-dependent weight matrix. $R(s)$ is assumed to be composed of rational functions of the Laplace operator $s$. Let us introduce a new input vector

$$\bar{v} = R(s)\bar{u} \tag{23}$$

Substituting Eq. (23) into Eq. (22) and using Parseval's theorem, we transform Eq. (22) into the equation

$$J = \int_0^{+\infty} \{\bar{y}^T Q \bar{y} + \bar{v}^T \bar{v}\}\, dt \tag{24}$$

The optimal control law, $\bar{v}_{\text{opt}}$, is obtained for the cost of Eq. (24) by LQ regulator theory. Then from Eq. (23), the original input vector $\bar{u}$ is obtained as

$$\bar{u} = R(s)^{-1}\bar{v}_{\text{opt}} \tag{25}$$

Let the transfer function of the linear system, which is the actuator or engine, be $G(s)$. Note that the LQ problem is solved for the augmented system $G(s)R(s)^{-1}$. Although the FDOR was originally designed for CT systems, it is applied to the DT systems here. Simulations prove the usefulness of its DT version.

The imaginary actuator dynamics are represented by the following pulse transfer function:

$$\delta_G(k) = \{p_0/(z - e_0)\}\delta_{Gc}(k) \tag{26}$$

where $e_0 = \exp(-1/T_{\max})$, $p_0 = 1 - e_0$, $z$ is the shift operator, $k = t/\Delta t$, and $\Delta t$ is a sampling rate. The compensator that corresponds to the inverse of weight in the frequency cost function is chosen as

$$R(z)^{-1} = p_1/(z - e_1) \tag{27}$$

where $e_1$ is a proper constant ($0 < e_1 < 1$) and $p_1 = 1 - e_1$. By applying the DT optimal regulator theory to the augmented system composed of the actuator and compensator, the optimal control law is obtained as

$$v'(k) = K_{1a}\delta_G(k) + K_{2a}\delta_{pr}(k) + v_c(k) \tag{28}$$

where $K_{1a}$ and $K_{2a}$ are optimal feedback gains, $v_c(k)$ is an external input vector, and $\delta_{pr}(k)$ is an output vector of the compensator. The FDOR is put in the proportional controller, and in order to eliminate steady-state error, the feedforward compensator is added. Given a proportional gain $K_{pa}$, a feed-

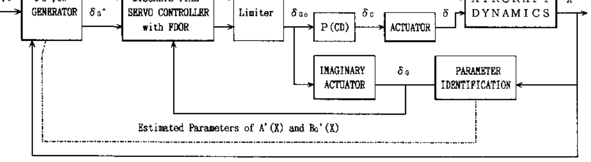


**Fig. 1 Block diagram of the RFCS.**

**Table 1 Simulated cases**

| Case no. | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Failure or normal? | Normal and failure A | Failure A | Failure B | Failure C |
| Control? | No | Yes | Yes | Yes |
| Restructure? | No | Yes | No | Yes |
| Initial condition | *IC*1 | *IC*1 | *IC*2 | *IC*1 |
| Trim point | — | *TP*1 | *TP*2 | *TP*1 |
| Figure no. | 2 | 3 | 4 | 5 |
| Stable/trimmed? | Normal: Yes<br>Failure: No | Yes | Yes | Yes |

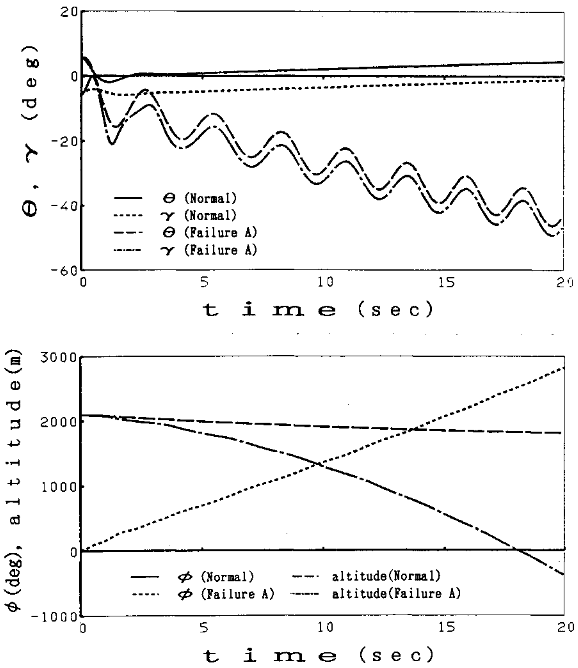


**Fig. 2 Time responses without control (case 1: free responses).**

forward compensator $p_2/(z-e_2)$, and its gain $K_{3a}$, the generic control inputs are represented by

$$\delta_{Gc}(k) = \{p_1/(z-e_1)\}v'(k) + \{K_{3a}p_2/(z-e_2)\}\delta_G^*(k) \quad (29)$$

In Eq. (29) $\delta_G^*(k)$, which is given by Eq. (20), is a reference input vector for $\delta_G(k)$. And $v'(k)$ and $K_{3a}$ are, respectively,

$$v'(k) = (K_{1a} - K_{pa})\delta_G(k) + K_{2a}\delta_{pr}(k) + K_{pa}\delta_G^*(k) \quad (30)$$

$$K_{3a} = 1 - K_{1a}/(1-K_{2a}) \quad (31)$$

The dynamics of the feedforward compensator are made slow enough for the feedforward inputs not to affect the feedback inputs in the transient state. Finally, command actual inputs $\delta_c$ are obtained by Eqs. (29) and (10).

The servo controller for the engine is designed in the same way, except that the reference input is given by Eq. (21) and does not need the control distributor. Figure 1 shows the block diagram of the RFCS.

## Simulation

Computer simulation was conducted using the 6-DOF nonlinear aircraft model (given in the Appendix) to demonstrate the performance of the RFCS. The aircraft is an F-14 class fighter and has seven control surfaces.

The state equations are given by Eqs. (1–4). The outputs and reference outputs are chosen as $Y=[u,\alpha,\theta,\phi]^T$ and $Y^*=[u^*,\alpha^*,\theta^*,\phi^*]^T$, respectively, where

$$Y^* = Y^*(\infty) + \mathrm{diag}\{\exp(-0.5t), \quad \exp(-3t) \quad \exp(-2t), \quad \exp(-3t)\}[Y(0) - Y^*(\infty)]$$

and

$$Y^*(\infty) = [220, 0.1, 0.1, 0]^T \quad (\mathrm{TP1})$$

or

$$[220, 0.1, 0.5236, 1.047]^T \quad (\mathrm{TP2})$$

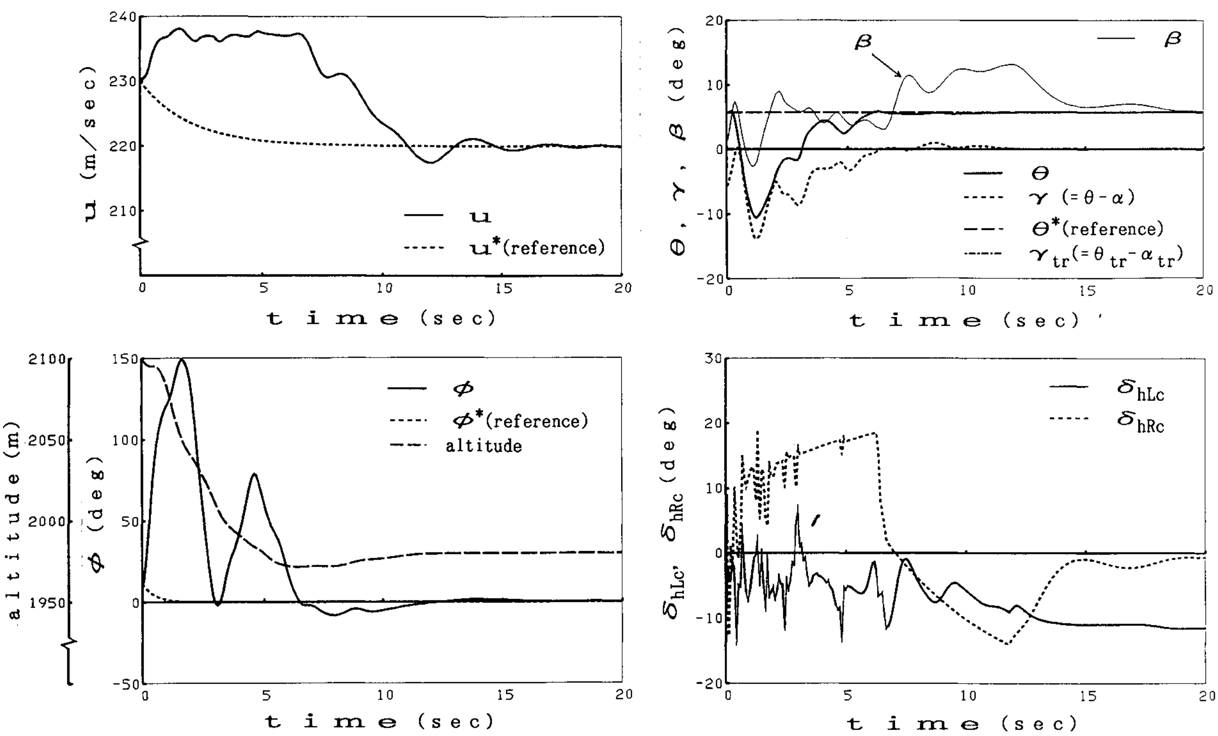


**Fig. 3 Time responses with restructure (case 2: failure A).**

Numerical data on the nominal aircraft configuration are the following: mass $m = 22{,}695$ kg, wing area $S = 48.77$ m$^2$, wing span $b = 19.2$ m, mean aerodynamic chord $c = 2.76$ m, moments of inertia $I_X = 67{,}790$ kgm$^2$, $I_Y = 427{,}348$ kgm$^2$, $I_Z = 476{,}564$ kgm$^2$, product of inertia $I_{XZ} = 0$ kgm$^2$, wing sweepback angle = 20 deg, wing taper ratio = 0.33, and acceleration of gravity $g = 9.8$ m/s$^2$. For the actuator time constants, $\Lambda^{-1} = \mathrm{diag}\{0.05, 0.05, 0.04, 0.04, 0.033, 0.033, 0.05\}$(s), $T_{\max} = 0.05$ s, and engine time constant $T_e = 1$ s. For a nominal flight condition, altitude = 2100 m, density of atmosphere $\rho = 0.9965$ kg/m$^3$, air speed $V = 221$ m/s (Mach 0.66).

The CD matrix $P$ is chosen as

$$P = \begin{bmatrix} 0.4 & 1.0 & 0.4 & 0.0 \\ 0.4 & 1.0 & -0.4 & 0.0 \\ 0.2618 & 0.5 & 0.2618 & 0.0 \\ 0.2618 & 0.5 & -0.2618 & 0.0 \\ -0.2 & -0.3 & 0.3 & 0.0 \\ -0.2 & -0.3 & -0.3 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.5236 \end{bmatrix} \tag{32}$$

The fixed parameters of the CT controller are $G_1 = -10$, $G_2 = [-225I_2, -30I_2]$. The parameters of the servo controller are $e_0 = 0.819$, $p_0 = 0.181$, $e_1 = 0.8$, $p_1 = 0.2$, $e_2 = 0.998$, $p_2 = 0.002$, $K_{pa} = 3.0$, $K_{1a} = -1.208$, $K_{2a} = -3.597$ for the actuators, $e_0 = 0.99$, $p_0 = 0.01$, $e_1 = 0.9$, $p_1 = 0.1$, $e_2 = 0.998$, $p_2 = 0.002$, $K_{pe} = 5.0$, $K_{1e} = -1.308$, $K_{2e} = -4.795$ for the engine.

The parameters or conditions in the simulation are as follows. Displacement limits for the surfaces are $|\delta_i|_{\max} = 0.4$ rad for $i = hL$, $hR$, 0.2618 rad for $i = aL$, $aR$, 0.3 rad for $i = cL$, $cR$, and 0.5236 rad for $i = r$. Available thrust limits are $0 \le T_h \le 1.78 \times 10^5$ N. The updating intervals equal 0.01 s for the inputs to the actuators, 0.2 s for the input to the engine, 0.05 s for the estimated parameters of the aircraft, and 0.05 s for the control parameters. Initial conditions are $X(0) = [230, 0.2, 0, 0.1, 5, 0, 0, 0.2]^T$ (*IC*1) or $X(0) = [220, 0.1, 0, 0.1, 0, 0, 0, 0]^T$ (*IC*2), where $\alpha(0)$ is shown instead of $w(0)$. Initial estimates equal the nominal parameters of the normal aircraft at the trim point, $X_{tr} = [220, 0.1, 0, 0.1, 0, 0, 0, 0]^T$, $U_{tr} = [-0.15596, -0.15596, 0.0081124, 0.0081124, -0.064657, -0.064657, 0.0, 0.11177 \times 10^6]^T$, where $\alpha_{tr}$ is shown instead of $w_{tr}$.

The following failures are considered:

*Failure A:* Half the right wing (the part of the right wing corresponding to $b/4 \le y \le b/2$) is broken off and the effectiveness of the right horizontal tail is reduced by 50%.

*Failure B:* The left horizontal tail is stuck at $-0.1$ (rad) and the effectiveness of the rudder is reduced by 50%.

*Failure C:* The effectiveness of the right horizontal tail is reduced by 23% and that of the right aileron becomes zero. It is assumed that the effectiveness of the right aileron becomes half of the nominal one in failure A.

In parameter identification, the least-squares method is modified so that the trace of the gain matrix cannot become too small, i.e., the matrix is reset if the trace is less than a certain value.

This RFCS deals with quite a large number of parameters and requires a considerable amount of computation. Therefore, in implementation, high-performance computers may be required. In addition, the time for interface operations such as A/D, D/A conversion is not negligible. The delay caused by computation and interface operations may affect the performance of the RFCS. In order to verify robustness against the delay, in the simulation control inputs are applied to the aircraft with one sampling period delay compared with regular digital control laws. The delay in parameter estimation is also evaluated in the same manner.

**Simulation Results**

The following cases summarized in Table 1 have been simulated. In the table "No" in the "Control?" row means a free flight where the control effectors are fixed at the trim values. "No" in the "Restructure?" row means that the parameters of the control laws are not updated, i.e., the nominal controller is used. In the figures, $\gamma$ is the flight-path angle ($= \theta - \alpha$) and the subscript $tr$ denotes the value at the target trim point.

*Case 1*

As seen from Fig. 2, the time responses of the normal aircraft show the typical longitudinal (and lateral-directional)

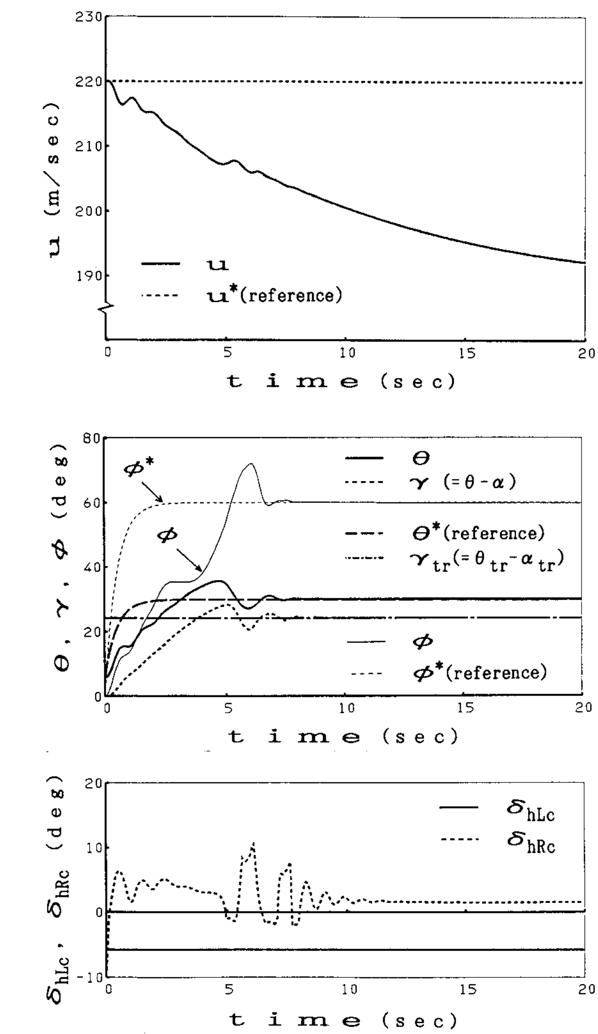


**Fig. 4 Time responses without restructure (case 3: failure B).**

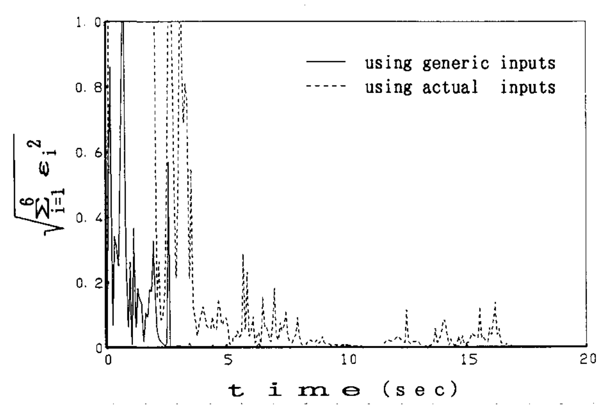


**Fig. 5 Time histories of identification error (case 4: failure C).**

modes. On the other hand, the figure shows that the impaired aircraft loses stability with respect to the roll motion. In fact, it falls, rolling at the rate of about 140 deg/s, and finally crashes to the ground about 18 s after the failure occurs.

*Case 2*

The results of Fig. 3 illustrate the effects of restructuring the controller. In this failure case, where it is confirmed that the aircraft cannot be trimmed and goes down without restructure, it is stabilized by the restructured controller. It takes about 15 s to trim the aircraft at the desired trim point. The time response of the roll angle indicates that the aircraft rolls 150 deg before being trimmed at $\phi = 0$ deg. Since the sideslip angle $\beta$ is not controlled, it converges to not zero, but about 6 deg. The amount of altitude indicates that the altitude loss is about 120 m. In failure A, it is confirmed that the linear RFCS fails to trim the aircraft. The linear controller is designed in the same way as the nonlinear one, except that the control law and identification model are based on the linear aircraft model.

*Case 3*

In this case, the aircraft that suffers failure B is commanded to complete the maneuver of 30-deg pitching up and 60-deg rolling from the level trim flight. Time responses are shown in Fig. 4 for the case without restructure. They indicate that the control objective can be attained by the unrestructured controller, as well as the restructured one, in spite of the fact that such a large maneuver is required. The results imply that the nominal controller is quite robust against the variation of the aircraft dynamics. As for the air speed, the available thrust is not enough for the aircraft to reach the specified speed within 20 s.

*Case 4*

This case shows how effective using generic inputs is in parameter identification. Figure 5 depicts the time histories of identification errors, which are defined by

$$\epsilon = \{\dot{X} - \hat{A}(X) - \hat{B}_G(X)U_G\}/(s+30) \tag{33}$$

The value of

$$\left(\sum_{i=1}^{6} \epsilon_i^2\right)^{1/2}$$

greater than 1 is truncated. Two cases, with and without generic inputs in identification, are compared. Failure C is considered here. As seen from the figure, while it takes the errors about 17 s to converge in the case using actual inputs, it takes about 3 s in the case using generic inputs. This is because the number of parameters in Eqs. (6) and (7), which is 157, is 29% smaller than that in Eqs. (1) and (3), which is 112. Since the equation of motion for $v$ is not used in the design, the parameters in the equation are not counted. Incidentally, it was confirmed in simulation that the RFCS using actual inputs cannot accommodate failure A.

Finally, let us mention the RFCS using three generic inputs. Since the number of the outputs, $[Y_1, Y_2^T]^T$, is three, the control law, Eq. (15), can also be determined for three generic inputs, such as $\delta_G = [\delta_{\text{lng}1}, \delta_{\text{lng}2}, \delta_{\text{roll}}]^T$. In fact, simulation was conducted for a number of failure cases using three generic inputs and the same CD matrix as Eq. (32), except that the fourth column was removed and the element $P(7,3)$ was 0.5236. In most cases, the results are as good as those obtained using four generic inputs. However, some failures such as failure A or control surface jam cannot be accommodated by three generic inputs. In this type of situation, the rudder angle exists in proportion to the aileron angles. Such a constraint may have an unfavorable effect on roll control.

## Conclusions

In this paper, a digital restructurable flight control system is designed by constructing a discrete-time servo controller for the actuators and engine. The reference inputs to the servo controller are given by the continuous-time control law that achieves the control objective. Simulation results show that the control system is able to accommodate failures of not only the control effectors, but also the airframe, which may considerably change stability or dynamical characteristics. Although the aircraft goes through large motions during the transient period, the control system works well. Therefore, it is effective in situations where nonlinearity of motion is rather large. Using generic inputs in parameter identification reduces the number of parameters and identification time. It means that while taking advantage of the functional redundancy of many actual inputs, the restructurable flight control system can complete failure identification efficiently. Since quick identification is a key factor in failure accommodation, it is important to use generic inputs. It was found through simulation that the nominal controller is quite robust in some failure cases. However, restructuring the flight control system ensures more safety of flight for impaired aircraft, especially for severely impaired ones.

## Appendix

Six-degree-of-freedom equations of motion are

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -g\sin\theta \\ g\cos\theta\sin\phi \\ g\cos\theta\cos\phi \end{bmatrix} + \begin{bmatrix} vr - wq \\ wp - ur \\ uq - vp \end{bmatrix} + \frac{P_{dy}}{m}\begin{bmatrix} C_x(\alpha) \\ C_y(\alpha,\beta) \\ C_z(\alpha) \end{bmatrix} + \frac{\rho VSb}{4m}\begin{bmatrix} 0 \\ C_{yp}(\alpha)p + C_{yr}(\alpha)r \\ 0 \end{bmatrix} + \frac{P_{dy}}{m}\begin{bmatrix} C_{x\delta}(\alpha)^T \\ C_{y\delta}(\alpha)^T \\ C_{z\delta}(\alpha)^T \end{bmatrix}\delta + \frac{1}{m}\begin{bmatrix} T_h \\ 0 \\ 0 \end{bmatrix} \tag{A1–A3}$$

and

$$E_1\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = E_2\begin{bmatrix} pq \\ qr \\ rp \end{bmatrix} + E_3\begin{bmatrix} p^2 \\ q^2 \\ r^2 \end{bmatrix} + P_{dy}\begin{bmatrix} bC_l(\alpha,\beta) \\ cC_m(\alpha,\beta) \\ bC_n(\alpha,\beta) \end{bmatrix} + \frac{\rho VSb^2}{4}\begin{bmatrix} C_{lp}(\alpha)p + C_{lr}(\alpha)r \\ (c/b)^2 C_{mq}(\alpha)q \\ C_{np}(\alpha)p + C_{nr}(\alpha)r \end{bmatrix} + P_{dy}\begin{bmatrix} bC_{l\delta}(\alpha)^T \\ cC_{m\delta}(\alpha)^T \\ bC_{n\delta}(\alpha)^T \end{bmatrix}\delta \tag{A4–A6}$$

along with

$$\dot{\theta} = q\cos\phi - r\sin\phi \tag{A7}$$

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta \tag{A8}$$

where $\beta$ $[=\sin^{-1}(v/V)]$ is the sideslip angle, $V[=(u^2+v^2+w^2)^{1/2}]$ is the air speed, and $C_x(\alpha)$, $C_l(\alpha,\beta)$, and so on are nondimensional aerodynamic forces or moments, which have the form of the polynomial functions of $\alpha$ or $\beta$, such as

$$(P_{dy}/m)C_x(\alpha) = (P_{dy}/m)\sum_{i=0}^{k} C_{xi}\alpha^i = \sum_{i=0}^{k}(C_{xi}\rho S/2m)(V^2\alpha^i) \tag{A9}$$

or

$$(P_{dy}/m)C_y(\alpha,\beta) = (P_{dy}/m)\sum_{i=0}^{k}\sum_{j=0}^{h} C_{yij}\alpha^i\beta^j = \sum_{i=0}^{k}\sum_{j=0}^{h}(C_{yij}\rho S/2m)(V^2\alpha^i\beta^j) \tag{A10}$$

$P_{dy} = \rho V^2 S/2$. The true values of $C_{xi}$, $C_{yij}$, and so on are determined using the data in Ref. 11. In Eqs. (A9) and (A10), $k=2$, $h=2$ for the identification model and $k=8$, $h=4$ for the actual aircraft model. The underlined coefficients, which are regarded as constant, are identified. In Eqs. (A4–A6), $E_1$, $E_2$, and $E_3$ are matrices whose elements are given by moments and products of inertia.

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